

1. Určete velikost tělivity, kterou na elipse  $e$  vytíná přímka  $p$ :

a)  $e: \frac{(x+3)^2}{25} + \frac{(y-1)^2}{16} = 1$ ,  $p: 4x + 5y - 13 = 0$

$$4x = 13 - 5y$$

$$x = \frac{13 - 5y}{4}$$

$$x+3 = \frac{13-5y}{4} + 3 = \frac{25-5y}{4} = \frac{5(5-y)}{4}$$

$$\frac{5^2(5-y)^2}{4^2 \cdot 25} + \frac{(y-1)^2}{16} = 1 \quad | \cdot 16$$

$$25 - 10y + y^2 + y^2 - 2y + 1 = 16$$

$$2y^2 - 12y + 10 = 0$$

$$y^2 - 6y + 5 = 0$$

$$D = 36 - 20 = 16$$

$$y_{1/2} = \frac{6 \pm 4}{2} = \begin{cases} 5 \\ 1 \end{cases}$$

$$x_1 = \frac{13 - 25}{4} = -3$$

$$K = [-3; 5]$$

$$x_2 = \frac{13 - 5}{4} = 2$$

$$L = [2; 1]$$

$$|KL| = \sqrt{25 + 16} = \sqrt{41}$$

b)  $e: \frac{x^2}{5} + \frac{(y+3)^2}{20} = 1$ ,  $p: 4x + y + 3 = 0$

$$y = -3 - 4x$$

$$y+3 = -3 - 4x + 3 = -4x$$

$$\frac{x^2}{5} + \frac{(-4x)^2}{20} = 1 \quad | \cdot 20$$

$$4x^2 + 16x^2 = 20$$

$$x^2 = 1$$

$$x = \pm 1$$

$$x_1 = 1 \quad y_1 = -3 - 4 = -7 \quad K = [1; -7]$$

$$x_2 = -1 \quad y_2 = -3 + 4 = 1 \quad L = [-1; 1]$$

$$|KL| = \sqrt{4 + 64} = \sqrt{68} = 2\sqrt{17}$$

c)  $e: \frac{(x-1)^2}{4} + \frac{(y+2)^2}{3} = 1$ ,  $p: y + 2 = 0$

$$y = -2 \quad y+2 = -2+2=0$$

$$\frac{(x-1)^2}{4} + \frac{0}{3} = 1 \quad | \cdot 4$$

$$x^2 - 2x + 1 - 4 = 0$$

$$x^2 - 2x - 3 = 0$$

$$D = 4 + 12 = 16$$

$$x_{1/2} = \frac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \end{cases}$$

$$K = [3; -2]$$

$$L = [-1; -2]$$

$$|KL| = \sqrt{16 + 0} = 4$$

d)  $e: \frac{(x+1)^2}{9} + \frac{(y-2)^2}{25} = 1$ ,  $p: x = -7 + 3t$

$$x+1 = -7 + 3t + 1 = 3t - 6 = 3(t-2)$$

$$\frac{3^2(t-2)^2}{9} + \frac{y^2}{25} = 1 \quad \underline{y = -3 + 5t}$$

$$y-2 = -3 + 5t - 2 = 5(t-1)$$

$$t^2 - 4t + 4 + t^2 - 2t + 1 - 1 = 0$$

$$2t^2 - 6t + 4 = 0$$

$$t^2 - 3t + 2 = 0$$

$$D = 9 - 0 = 9$$

$$t_1 = 2 \quad x = -7 + 6 = -1 \quad K = [-1; 7]$$

$$y = -3 + 10 = 7$$

$$t_2 = 1 \quad x = -7 + 3 = -4 \quad L = [-4; 2]$$

$$y = -3 + 5 = 2$$

$$|KL| = \sqrt{9 + 25} = \sqrt{34}$$

$$t_{1/2} = \frac{3 \pm 1}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

2. Určete hodnotu parametru  $m$  tak, aby přímka  $p$  byla tečnou na elipsy  $e$ :

a)  $e: \frac{(x+2)^2}{1} + \frac{y^2}{2} = \frac{7+3m}{2}$ ,  $p: 5x - y + 1 = 0$   $y = 5x + 1$

$$2(x^2 + 4x + 4) + (25x^2 + 10x + 1) = 7 + 3m$$

$$27x^2 + 19x + 2 - 3m = 0$$

$$D = 19^2 - 4 \cdot 27(2 - 3m) = 0$$

$$324 - 216 + 324m = 0$$

$$324m = -108$$

$$m = -\frac{1}{3}$$

b)  $e: \frac{(x+2)^2}{4} + \frac{(y+4)^2}{5} = \frac{m}{10}$ ,  $p: x - y - 5 = 0$   $y = x - 5$

$$y + 4 = x - 5 + 4 = x - 1$$

$$\sqrt{(x^2 + 4x + 4) + 4(x^2 - 2x + 1)} = 2m$$

$$9x^2 + 12x + 24 - 2m = 0$$

$$D = 144 - 4 \cdot 9(24 - 2m) = 0$$

$$144 - 864 + 72m = 0$$

$$72m = 720$$

$$m = 10$$

c)  $e: \frac{x^2}{8} + \frac{y^2}{2} = \frac{m}{4}$ ,  $p: x + 2y - 1 = 0$

$$x = 1 - 2y$$

$$(1 - 2y)^2 + 4y^2 = 2m$$

$$1 - 4y + 4y^2 + 4y^2 - 2m = 0$$

$$8y^2 - 4y + 1 - 2m = 0$$

$$D = 16 - 4 \cdot 8(1 - 2m) = 0$$

$$16 - 32 + 64m = 0$$

$$64m = 16$$

$$m = \frac{1}{4}$$